Part (d)(i)

Game plan: The 28th of March is 7 days after the 21st, so use $t = 7$ and radians.

$$\begin{aligned} H(7) &= 4 \cdot 5 \sin\left(\frac{2\pi}{365}(7)\right) + 12 \\ &= 12 \cdot 540936 \ldots . \end{aligned}$$

Final Answer

12 · 54 daylight hours, to 2 decimal places.

Part (d)(ii)

Game plan: Find the two times when daylight is exactly 8 hours. Then choose the whole days strictly between them.

Step 1: Find the boundary angles

Let $\theta = 2\pi t/365$. At the boundary:

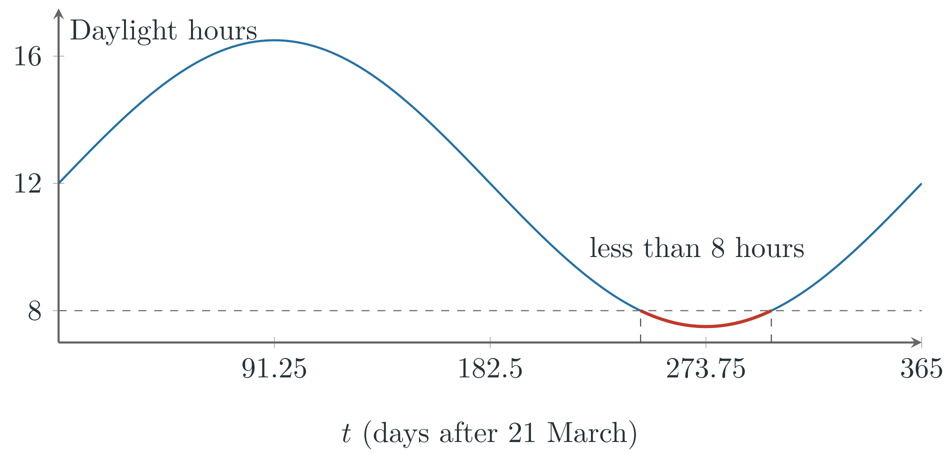
$$\begin{aligned} 4 \cdot 5 \sin\theta + 12 &= 8 \\ \sin\theta &= -\frac{8}{9}. \end{aligned}$$

The positive reference angle is $\alpha = \sin^{-1}(8/9) = 1 \cdot 094914\ldots$ radians. Sine is negative in the third and fourth quadrants, giving:

$$\theta_1 = \pi + \alpha, \qquad \theta_2 = 2\pi - \alpha.$$

Step 2: Convert the angles into days

$$\begin{aligned} t_1 &= \frac{365(\pi + \alpha)}{2\pi} = 246 \cdot 105\ldots, \\ t_2 &= \frac{365(2\pi - \alpha)}{2\pi} = 301 \cdot 394\ldots . \end{aligned}$$



The model has less than 8 hours for $246 \cdot 105\ldots < t < 301 \cdot 394\ldots$.

Step 3: Choose the first and last whole days

The first integer after the lower boundary is 247. The last integer before the upper boundary is 301. Ordinary rounding of both boundaries would give the wrong first day.

Final Answer

First day: $t = 247$. Last day: $t = 301$.