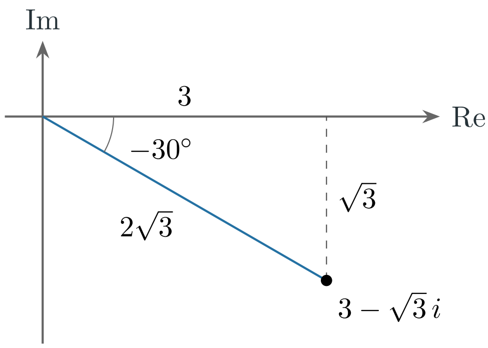
Part (b)

Game plan: Write the number in polar form, use de Moivre's theorem to raise it to the eighth power, then return to $a + bi$.

Step 1: Find the modulus and angle

Let $z = 3 - \sqrt{3}\,i$. Its modulus is $r = \sqrt{3^2 + (-\sqrt{3})^2} = \sqrt{12} = 2\sqrt{3}$. The point lies below the positive real axis, so choose a negative angle.

$$\tan|\theta| = \frac{\sqrt{3}}{3} \quad \Longrightarrow \quad \theta = -30^\circ.$$



Step 2: Raise the modulus and multiply the angle

De Moivre's theorem — **Logbook p. 20**

$$[r(\cos\theta + i\sin\theta)]^n = r^n(\cos n\theta + i\sin n\theta).$$

$$\begin{aligned} z^8 &= (\sqrt{12})^8\big(\cos(-240^\circ) + i\sin(-240^\circ)\big) \\ &= 20\,736\big(\cos 120^\circ + i\sin 120^\circ\big). \end{aligned}$$

We added 360° to the angle. This makes one full turn, so it represents the same point.

Step 3: Write the answer in rectangular form

$$\begin{aligned} z^8 &= 20\,736\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) \\ &= -10\,368 + 10\,368\sqrt{3}\,i. \end{aligned}$$

Final Answer

$\boldsymbol{-10\,368 + 10\,368\sqrt{3}\,i.}$