

Part (b)

Game plan: We turn the proportional rate of decrease into a differential equation, solve it, then use the known height after 45 seconds to find the constant.

Step 1: Write the differential equation

Let $k > 0$ be the constant of proportionality. Because the height is decreasing,

$$\frac{dx}{dt} = -kx.$$

Step 2: Separate and integrate

Logarithmic integral	Logbook Page 26
$\int \frac{1}{x} dx = \ln x + C.$	

Since $x > 0$ while water remains in the tank,

$$\begin{aligned} \frac{dx}{x} &= -k dt, \\ \ln x &= -kt + C. \end{aligned}$$

At $t = 0$, $x = H$, so $C = \ln H$. Therefore

$$\ln \left(\frac{x}{H} \right) = -kt, \quad x = He^{-kt}.$$

Step 3: Use the height after 45 seconds

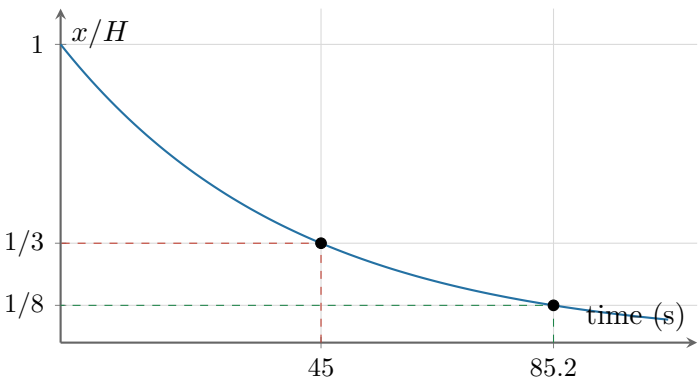
At $t = 45$, $x = H/3$. Substitute into the logarithmic form:

$$\begin{aligned} \ln \left(\frac{1}{3} \right) &= -45k, \\ k &= \frac{\ln 3}{45}. \end{aligned}$$

Step 4: Find when the height is $H/8$

$$\begin{aligned} \ln \left(\frac{1}{8} \right) &= -kt, \\ t &= \frac{\ln 8}{k} = 45 \frac{\ln 8}{\ln 3} \\ &= 85.175 \dots \text{ s.} \end{aligned}$$

The graph shows the height as a fraction of its initial value.



Final Answer
$t \approx 85.2$ s after the water starts draining.