

Part (d)

Game plan: Focus on the quarter-circle end. The straight-sided parts are already exact trapezia, so all the error comes from approximating the curve.

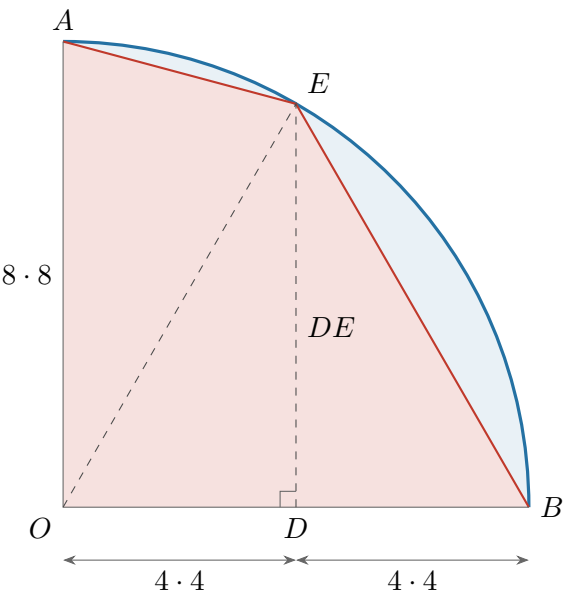
Step 1: Find the radius and strip width

Let O be the circle's centre, directly below A on BC . The radius is $OA = OB = 8 \cdot 8$ m. There are two equal intervals from O to B , so each has width $8 \cdot 8/2 = 4 \cdot 4$ m.

Step 2: Find the missing height on the arc

At the middle point D of OB , let the perpendicular meet the arc at E . The right triangle ODE has hypotenuse $OE = 8 \cdot 8$ and base $OD = 4 \cdot 4$.

$$\begin{aligned} DE &= \sqrt{8 \cdot 8^2 - 4 \cdot 4^2} \\ &= \sqrt{58 \cdot 08} = 7 \cdot 621023 \dots \end{aligned}$$



The straight chords leave out the area beside the arc.

Step 3: Compare the exact curved area with its estimate

The actual end area is a quarter-circle:

$$A_{\text{exact}} = \frac{1}{4} \pi (8 \cdot 8)^2 = 60 \cdot 8212 \dots \text{ m}^2.$$

Trapezoidal rule

Logbook p. 12

$$A \approx \frac{h}{2} [y_1 + y_n + 2(y_2 + \dots + y_{n-1})].$$

There are three heights here: $8 \cdot 8$, $\sqrt{58 \cdot 08}$, and 0, with spacing $h = 4 \cdot 4$.

$$\begin{aligned} A_{\text{estimate}} &= \frac{4 \cdot 4}{2} [8 \cdot 8 + 0 + 2\sqrt{58 \cdot 08}] \\ &= 52 \cdot 8925 \dots \text{ m}^2, \\ \text{Error} &= 60 \cdot 8212 \dots - 52 \cdot 8925 \dots \\ &= 7 \cdot 9287 \dots \text{ m}^2. \end{aligned}$$

Final Answer

Liam's estimate is **8 m² too small**, to the nearest square metre.